Applications

1. a. \( \frac{28}{50} = \frac{14}{25} \), or 56% 
   
   b. It should come up heads about 250 times, or half of 500. It will most likely not be exactly 250 heads in 500 tosses, but it is unlikely to be far from 250 heads.

2. He tosses each day for about 6 years. 
   \( 6 \times 365 \text{ days} = 2,190 \text{ days} \) (2,191 days; some students might add a day or two for leap years) for which Kalvin will toss a coin. You would expect him to toss heads and eat Cocoa Blast about half of those days, or about 1,095 days.

3. With only five trials, you cannot be certain. Kalvin should toss the coin many more times if he wants to find out whether or not the coin is fair. In fact, the probability of five consecutive heads is \( \frac{1}{32} \approx 3\% \).

4. The chances are \( \frac{1}{2} \), or 50%. If a coin turns up heads three times in a row, it is not more likely to turn up tails the next time, nor is it more likely to be heads again. This can be confusing for students because they expect the average to be about 50% in the short run. Experimental results are about results in the long run.

5. It is possible, but unlikely. Each time a coin is tossed it can land heads up, so 20 heads in a row is possible. However, there are many more possible combinations of 20 coin tosses that are not all heads, so 20 heads is very unlikely. The chance of getting 20 heads in a row is about 0.000001, that is, about 1 chance in a million.

6. a. Answers will vary based on the experiments conducted in class. A group of students that found 20 ends in the 50 tosses should argue for a number about 7 times as large (146 ends and 219 sides in a year, \( 50 \times 7 = 350 < 365 \)).
   
   b. Answers will vary based on the experiments conducted in class. In the example from part (a), 40% of the tosses resulted in ends, so this should happen about 12 times a month (40% of 30), or 146 times a year (40% of 365).

7. The pawn is more likely to land on its side, because it is better to base a prediction on 100 tosses than on 5 tosses. It gives you even more information if you combine the data.

8. Kalvin should use the small cup and eat Cocoa Blast when it lands on its side. This is because the large cup landed on its side about 73% of the time in his experiments while the small cup landed on its side 78% of the time. Some students may argue that the number of trials was not sufficiently large with the large cup and so the probabilities may be even closer than they appear.

9. Red and blue are like heads and tails. Each bag is like a coin. Red and blue are equally likely in each bag, just as heads and tails are on each coin.

10. Adsila should not agree. The probability of getting at least one head is 75%. Students can determine this by considering the possibilities, or by referring to their data from Problem 1.3.

11. The results are probably not equally likely. It is more likely that a friend or family member would call at 9:00 P.M. than someone else.

12. Depending on the season, the results are probably not equally likely. Students should have a good idea of whether the local temperature is more likely to be 30°, higher than 30°, or under 30°F.

13. The results are not equally likely. The spinner landing on red is more likely than the spinner landing on yellow or on blue, because the red takes up \( \frac{1}{2} \) of the circle’s area, whereas the yellow and blue only take up \( \frac{1}{4} \) of the circle’s area. Students may want to experiment with this spinner or analyze the spinner with their angle ruler.
14. The results are probably not equally likely, as the size and layout of the town or city would affect the number of accidents. (To find out for your city, you would need to know the average number of accidents that occur on a given day. The average might be different for a Monday than a Saturday.)

15. The results are equally likely. Each standard deck of playing cards contains exactly 13 spades, 13 hearts, 13 diamonds, and 13 clubs. Thus, the chances of drawing any one particular suit are \( \frac{1}{4} \).

16. There are three possible results—choosing a red block, a blue block, or a green block. The results are not equally likely. There are more blue blocks in the bag, so the chances of drawing a blue block are greater than the chances of drawing a red or green block.

17. There are two possible results—you succeed in stealing second base, or you are out. These results are probably not equally likely. Their chances depend on the person’s skill and experience playing baseball and stealing bases.

18. Possible answers:
   a. It will be 250°F in Texas on the first day of January.
   b. You guess the right answer on a multiple choice question with four options.
   c. A quarter will land heads up when it is tossed.
   d. It will snow during a week of winter in New Hampshire.
   e. When you choose a letter at random from the letters A, B, C, D, and F, you choose a consonant.
   f. The sun will set tonight

19. a. \( \frac{9}{50} \), 18%
   b. \( \frac{23}{50} \), 46%
   c. \( \frac{29}{50} \), 58%
   d. Answers will vary. Possible answers: 16 (the result of 100 ÷ 6); students may double the results from the given table and answer “18 times.”
   e. She can expect about 500 odd numbers, since odd and even are equally likely. Students may scale the results from this table and respond “460 times.”

20. a. Answers will vary. Possible answers:
   \( \frac{1}{5} \), \( \frac{9}{50} \), \( \frac{1}{4} \)
   b. Answers will vary. Possible answers:
   \( \frac{12}{40} \), \( \frac{13}{40} \), \( \frac{14}{40} \)

21. B
22. G
23. C

24. a. Answers will vary. Students should describe putting all the factors, 1, 2, 3, 6, 7, 14, 21, and 42 on pieces of paper, then repeating several trials to make the experiment. Sample for 20 trials: 1, 3, 21, 42, 2, 7, 6, 2, 42, 21, 3, 3, 6, 21, 7, 7, 14, 42, 1, 42, and 21. This trial leads to a probability of \( \frac{9}{20} \) even factors.
   
   Note: The theoretical probability is \( \frac{1}{2} \).
   b. Answers will vary. Using the sample data from part (a), the probability for prime factors is \( \frac{8}{20} \). Note: The theoretical probability is \( \frac{3}{8} \).

25. a. 40%, since \( \frac{2}{5} = \frac{40}{100} \), or since \( 2 \div 5 = 0.40 \)
   b. 30%, since \( \frac{3}{10} = \frac{30}{100} \), or since \( 3 \div 10 = 0.30 \)
   c. 60%, since \( \frac{3}{5} = \frac{60}{100} \), or since \( 3 \div 5 = 0.60 \)
26. No, a tornado is more likely to occur somewhere in Florida.

27. Yes, tornados are equally likely to occur in Arkansas and Pennsylvania.

28. No, a tornado is more likely to occur somewhere in Texas.

29. No. Although the data show that more tornadoes strike Montana than Massachusetts, this does not mean that a resident of Montana is more likely to experience a tornado than a resident of Massachusetts. For example, consider that Massachusetts is much smaller than Montana, and that many fewer people live in Montana.

**Extensions**

30. There are ten possibilities in all:

<table>
<thead>
<tr>
<th>First H in 1st Position</th>
<th>First H in 2nd Position</th>
<th>First H in 3rd Position</th>
<th>First H in 4th Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>HHTTT</td>
<td>THHTT</td>
<td>TTHHT</td>
<td>TTTTH</td>
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<tr>
<td>HTHTT</td>
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</tbody>
</table>

Possible explanation: I have organized my list in a way that helps me be sure I have all the possibilities. First, I put one H in the first position and moved the second H through the other four positions. Then I put the H in the second position and moved the other H into the third through the fifth positions. I continued this pattern until I had covered all the possibilities. I know I have not duplicated any because the first H changes position with each column.

31. a. Since the cup lands upright 5 times out of 50, Yolanda can expect to win about 1 out of 10 times. She can expect to lose about 9 out of 10 times.

b. Less; Yolanda would have to spend $10 to play 10 times. You should expect her to win once, giving her $5, so she would have spent $5 more than she won. Of course, this is only a good guess about what to expect. She may actually lose more money or win money.