Investigation 1

ACE Assignment Choices

Problem 1.1
Core 1–7
Other Connections 26–28, 30; Extensions 35, 36

Problem 1.2
Core 8–10, 14
Other Applications 11–13; Connections 29, 31; Extensions 37; unassigned choices from earlier problems

Problem 1.3
Core 15–25
Other Connections 32–34; Extensions 38, 39; unassigned choices from earlier problems

Adapted For suggestions about adapting Exercises 1–6, 8–10, and other ACE exercises, see the CMP Special Needs Handbook.

Connecting to Prior Units 29, 31: Moving Straight Ahead, Thinking With Mathematical Models; 32: Bits and Pieces II; 33, 38, 39: Covering and Surrounding; 34: Accentuate the Negative

Applications

1. a. (6, 1)  b. (−6, −4)  c. (−6, 0)
2. 13 blocks  3. 18 blocks
4. There are many 10-block routes, but there are exactly five possible halfway points: (−5, 0), (−4, −1), (−3, −2), (−2, −3), and (−1, −4).
5. Because there is only one possible route, there is only one possible halfway point: (−3, −2).
6. a. The art museum and the cemetery
   b. Possible answer: To get to the art museum, drive 6 blocks east, turn left, and go north 1 block. To get to the cemetery, drive 3 blocks east, turn right, and drive 4 blocks south.
7. a. The hospital is 4 blocks from the greenhouse. There are ten intersections on the map that are 4 blocks by car from the gas station: (1, 5), (0, 4), (1, 3), (2, 2), (3, 1), (4, 0), (5, 1), (6, 2), (7, 3), and (7, 5).
   b. (−2, 3) and (1, 5); (5, −1) and (2, −3). There is a third possibility with non-integer coordinates, but students do not need to find this one.
8. There are infinitely many possible pairs, including (2, 0) and (5, 2); (0, 2) and (3, 4); (0, −2) and (3, 0); and (2, −1) and (5, 1).
9. There are infinitely many possible vertices, including (0, 2), (3, 0), (4, −6) and (5, −1). Any one of the vertices in Question 8 will work.
10. B
11. B
12. There are many possible vertices, including (2, 3), (3, 6), (5, 7), (1, 4), (4, 5), (0, 2), (6, 4). (See the answer to Exercise 13.)
13. An infinite number of right triangles can be drawn. The third vertex can be located at any grid point on the line that goes through (0, 2) and (6, 4) (the line \( y = \frac{1}{3}x + 2 \)) or on the line that goes through \((-1, 5)\) and \((5, 7)\) (the line \( y = \frac{1}{3}x + \frac{16}{3} \)). Each of these lines is perpendicular to the segment connecting \((3, 3)\) and \((2, 6)\), so these lines create the right angle for the triangle. Some students may express this idea as follows: Imagine a line starting from one of the given points and at a right angle to the given side. Any point along that line can be the third vertex of the triangle.

14. Yes. Opposite sides have equal lengths and slopes.

Note: The slopes may be compared intuitively at this time. Students may say the distance between parallel lines is always the same, or they may use left/right, up/down language to express this idea. Others may find the actual slopes.

15. 3 units\(^2\)  
16. 4 units\(^2\)  
17. 2 units\(^2\)  
18. 2 units\(^2\)  
19. 3.5 units\(^2\)  
20. 5 units\(^2\)  
21. 5 units\(^2\)  
22. 2.5 units\(^2\)  
23. 1 unit\(^2\)  
24. 5.5 units\(^2\)  
25. 8.5 units\(^2\)

Methods used in Exercises 21–25 will vary. Students may subdivide a figure into smaller squares and triangles and add their areas. They might surround a figure with a rectangle and subtract the areas of the shapes outside of the figure from the rectangle’s area. For example, a square of area 4 units\(^2\) can be drawn around the shape in Exercise 23, and the area of the three 1 unit\(^2\) triangles can be subtracted, leaving an area of 1 unit\(^2\).

Connections

26. 8 blocks \(\times\) 150 m/block = 1,200 m
27. 12 blocks \(\times\) 150 m/block = 1,800 m
28. 750 m \(\div\) 150 m/block = 5 blocks. City Hall and the Stadium are 5 blocks, or 750 meters, apart by car. So are the Cemetery and the Animal Shelter, and the Art Museum and the Gas Station.
29. a. She probably found the slopes of all four sides. The slopes of any two adjacent sides are negative reciprocals of each other, so they are perpendicular line segments (in other words, all four angles were 90°).
   b. She probably found the slopes of all four sides. Because the slopes of opposite sides were the same, they were parallel. Because opposite sides of the quadrilateral were parallel, her figure was a parallelogram.
30. a. \((-2, -1)\)
   b. There are three ways to find the shortest route. For example, Cassandra could walk 2 blocks west and 1 block south.
   c. \((-1, 4)\)
   d. There are five ways to find the shortest route. For example, Aida could walk 1 block west and 4 blocks north.
   e. Figure out how many blocks east or west you have to go by comparing the \(x\)-coordinates of the two locations. Figure out how many blocks north or south you have to go by comparing the \(y\)-coordinates. The sum of these is the number of blocks in a shortest route.
31. a. Lines 1, 5, and 8; lines 3 and 6
   b. Lines 2 and 6; lines 3 and 2; lines 8 and 4; lines 1 and 4; lines 5 and 4
32. a. \(\frac{31}{2}\) units\(^2\)
   b. Answers will vary. Possible figure:

33. a. \(4\pi\), or about 12.56 units\(^2\)
   b. \(16 - 4\pi\), or about 3.43 units\(^2\)
34. a. (6, 0). It has the greatest $x$-coordinate.  
b. (−5, −5). It has the least $x$-coordinate.  
c. (−4, 6). It has the greatest $y$-coordinate.  
d. (0, −6). It has the least $y$-coordinate.

**Extensions**

35. Road maps are typically partitioned into square areas by consecutive letters running along the sides of the map and consecutive numbers running along the top and bottom. This system is similar to a coordinate grid system, but the letters and numbers do not refer to points; they refer to regions. For example, anything in the top-left square might be in region A-1.

36. Answers will vary. Students should include compass directions as well as distances and will need to decide where the distances are to be measured from, such as airports or city centers. For example: Starting at the airport at Grand Rapids, go south 47 mi to the airport at Kalamazoo. From Kalamazoo, go northeast 60 mi to the airport at Lansing. From Lansing, go southeast 80 mi to the airport at Detroit.

**For the Teacher** You may want to point out that pilots need more exact directions than north, south, east, or west because the actual direction may be a few degrees east or west of due north.

37. Possible answer: For each parallelogram, all four sides are the same length. A rhombus is the only parallelogram with perpendicular diagonals. Students may only say that squares—rhombi with right angles—have perpendicular diagonals. You may want to encourage them to look for non-square rhombi.

38. Each triangle has an area of 1 unit$^2$. They all have base length 1 unit and height 2 units.

39. Each triangle has an area of 3 units$^2$ because they all have base 3 units and height 2 units.

**Possible Answers to Mathematical Reflections**

1. Driving distances are the same as or greater than flying distances. If the two places do not lie on the same vertical or horizontal line, the flying distance is shorter because the car can’t travel in a straight line between them, but the helicopter can.

2. Note that “distance” is intentionally vague. Students encountered two types of distances in Euclid: driving and flying. The flying distance corresponds to straight-line distance on the plane. Flying distances can be estimated with a ruler. Calculating flying distances exactly requires using the Pythagorean Theorem, which students do not yet know. The driving distance between two landmarks is the sum of the positive differences of the $x$- and $y$-coordinates. In other words, the driving distance is the sum of the absolute value of the differences between the $x$- and $y$-coordinates.

3. Sometimes I just counted the units of area. Sometimes I subdivided the figure into smaller shapes like right triangles and rectangles, found the areas of the smaller shapes, and added them to get the large figure’s area. Sometimes I enclosed the figure in a rectangle, found the area of the rectangle, and subtracted the areas of the figures that were not part of the enclosed figure.